

Limnology – In-class exercises no 9

List of constants

Thermal expansivity	$\alpha(T) \approx 14 \cdot 10^{-6} \cdot (T [^{\circ}\text{C}] - 4 [^{\circ}\text{C}]) [\text{K}^{-1}]$
Kinematic viscosity	$\nu_{\text{water}} (5 [^{\circ}\text{C}]) = 1.52 \cdot 10^{-6} \text{ m}^2 \text{ s}^{-1}; \nu_{\text{air}} (15 [^{\circ}\text{C}]) = 14.5 \cdot 10^{-6} \text{ m}^2 \text{ s}^{-1}$
Molecular diffusivity	$D_{\text{Temp}} \approx 1.4 \cdot 10^{-7} \text{ m}^2 \text{ s}^{-1}; D_{\text{O}_2} \approx 1.6 \cdot 10^{-9} \text{ m}^2 \text{ s}^{-1}$
Specific heat of water	$c_{\text{pw}} = 4200 \text{ J kg}^{-1} \text{ K}^{-1}; \rho c_{\text{pw}} = 4.2 \cdot 10^6 \text{ J m}^{-3} \text{ K}^{-1}$
Density of air	$\rho_{\text{air}} = 1.23 \text{ kg m}^{-3}$
Von Karman constant	$\kappa = 0.41$ (non-dimensional)
Bottom drag coefficient	$C_{\text{D}} = 0.0010$ (non-dimensional)

1) Effects of processes on water column stability

Consider the following processes and explain the effect of the buoyancy flux:

(i) stabilizes water column (ii) causes convective turbulence (iii) has no effect

- (a) Warming of Lake Geneva water surface in winter when $T_{\text{Surf}} < 4 [^{\circ}\text{C}]$.
- (b) Warming of the water surface in the North Atlantic for $T_{\text{Surf}} < 4 [^{\circ}\text{C}]$.
- (c) Freezing of North Atlantic water.
- (d) Inflowing Jordan River water into the saline Dead Sea.
- (e) Decomposition of organic matter at the sediment surface of Lake Lugano.
- (f) Release of CH_4 - CO_2 bubbles from an anoxic sediment.
- (g) Reinjection of warm cooling-water into the hypolimnion. Cooling-water is typically cold lake water, which is warmed by cooling infrastructure / processes and released back to the lake.
- (h) The intrusion of geothermal water from sub-aquatic sources into the deepest zone of the Red Sea, which has a temperature of $\sim 58 [^{\circ}\text{C}]$ and a salinity of $\sim 320\text{‰}$.
- (i) Particle-laden Rhône River water entering Lake Geneva and diving deep as a turbidity-flow into the hypolimnion (e.g. after a thunderstorm).
What happens in summer when the Rhône water is $\sim 11 [^{\circ}\text{C}]$ and therefore much warmer than the cold deep-water of the hypolimnion? (*Hint: Ignore salinity, which is low*).
- (j) Injection of compressed air into the deepest layers of Lake Sempach artificially enhancing the seasonal deep-convective mixing in winter.

2) Double diffusion – heat flux

In the **Figure below**, you see a short temperature section containing **four well-mixed layers** of the double-diffusive staircase from Lake Kivu (there are more than 300 interface/layer pairs throughout the entire water column of $\sim 485 \text{ m}$ max depth).

- (a) Based on the profile: Is this a **finger-regime** or a **diffusive-regime**? What makes the water column stable?
- (b) Determine the molecular heat flux (W m^{-2}) through the interface at $\sim 189.2 \text{ m}$ depth. Does this value reasonably compare with the global geothermal heat flux?
- (c) In the inset of the figure, you can see the temperature fluctuations T' and you can estimate the temperature differences between sinking and rising plumes in the mixed layer. What density differences do these fluctuations correspond to?
- (d) Determine the buoyancy flux within the layer.

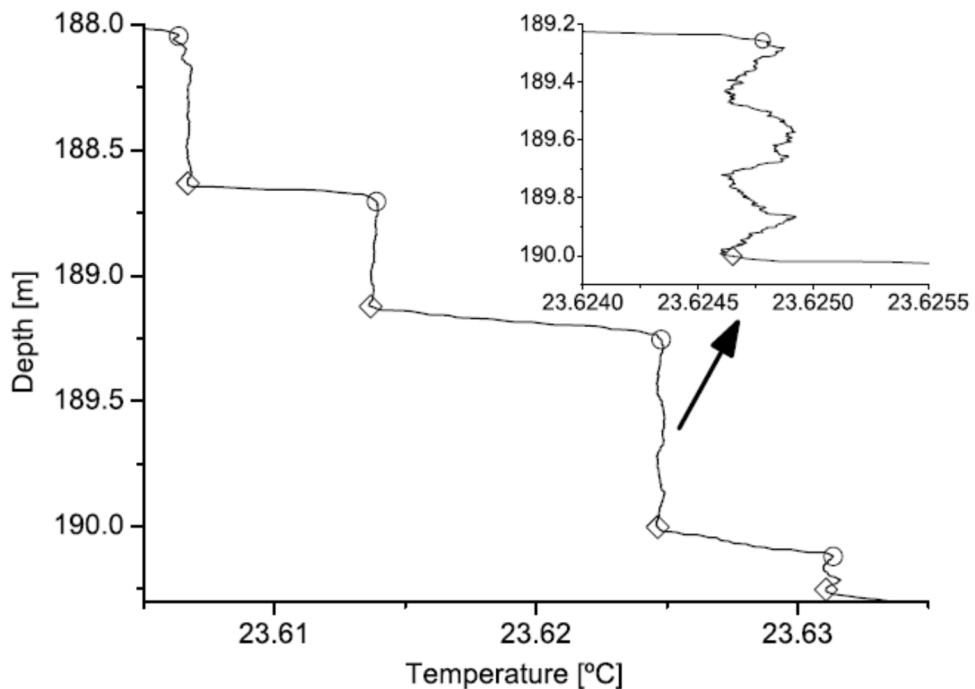
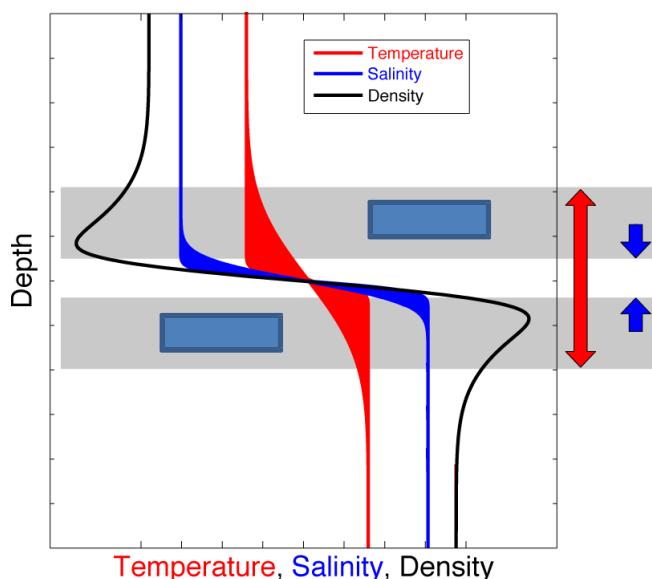


Figure A 2-m long section of the vertical temperature profile in the double diffusive staircase as observed in Lake Kivu (Eastern Africa). Sources: Newman (1976). Sommer et al (2014).

3) Double-diffusive induced boundary layer instability

In the figure below you see two completely homogenous layers, which are separated by an interface in which temperature and salinity change rapidly from the upper layer to the lower layer. To give you a practical sense: this realistic vertical stratification structure occurs in Lake Kivu and the two homogenous layers are typically ~ 80 cm thick. The transition from the upper to the lower mixed layer is indicated by the red arrow. The interface in between the layers, where temperature and salinity transit linear, is 10 cm thick and is marked by the two blue arrows.



(Hint: Note that in the solution of the Heaviside problem (Eq 3.12 Classnotes) the width of the transition of the error function is $2\delta \sim 2\sqrt{4Dt}$).

Assume a Heaviside step initially between the two layers

- How much time did it take to evolve the temperature profile shown? And the salinity profile? (approximately!)
- Explain the different behaviour of T and S in the interface.

Explain the density profile in the two grey zones.

- Are these two grey boundary layers stably or unstably stratified? Explain the reason.
- Explain whether you expect any flow dynamics/currents from this situation?
- How will this interface situation further develop in the next days, weeks, months? How long will it be alive?